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Cut Locus and Topology from Surface Point Data

A cut locus of a point p in a compact Riemannian manifold M is defined as the set of points where minimizing geodesics issued from p stop being minimizing. It is known that a cut locus contains most of the topological information of M. Our goal is to utilize this property of the cut loci to decipher the topology of M from a point sample. Recently it has been shown that Rips complexes can be built from a point sample P of M systematically to compute the Betti numbers, the rank of the homology groups of M. Rips complexes can be computed easily and therefore are favored over others such as restricted Delaunay, alpha, Čech, and witness complex. However, the sizes of the Rips complexes tend to be large. Since the dimension of a cut locus is lower than that of the manifold M, a sub-sample of P approximating the cut locus is usually much smaller in size and hence admits a relatively smaller Rips complex.

In this paper we explore the above approach for surfaces embedded in any high dimensional Euclidean space. We present an algorithm that computes a sub-sample P' of a sample P of a 2-manifold where P' approximates a cut locus. Empirical results show that the first Betti number of M can be computed from the Rips complexes built on these sub-samples. The sizes of these Rips complexes are much smaller than the one built on the original sample of M.